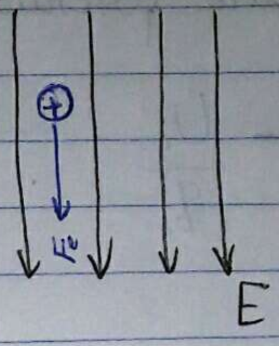
24 - Electric potential

$$W_{\text{done By } F_E} = \int_i^f \vec{F}_E \cdot d\vec{r}$$



$$W_E = \int_i^f q_0 \, \vec{E} \cdot d\vec{r} \quad J$$

- Electric potential energy (U)
- $\vec{F}_E = q_0 \cdot \vec{E}$ is a conservative force.
- $W_E = -\Delta U$ .... (*)

$W_E(1) = W_E(2)$

$W_E = \Delta K$

$$\Delta U = U_f - U_i = -W_E = -\int_i^f q_0 \, \vec{E} \cdot dr \quad J$$

- $U_\infty = 0$ .... (*)
- $U_f = -\int_\infty^f q \cdot \vec{E} \cdot d\vec{r}$ .... (*)

U_f = - work done by $\vec{E}$ in moving q_0 (from ∞ to f)

- Electric potential (V)

$$\frac{U_f}{q_0} - \frac{U_i}{q_0} = -\int_i^f E \cdot dr \qquad J/C$$

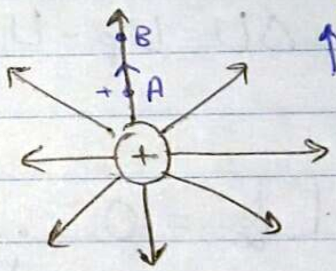
$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{r} \qquad \text{Volt}$$

$$\Rightarrow \frac{U_f}{q_0} = V_f = -\int_\infty^f \vec{E} \cdot dr \qquad \text{Volt}$$

$V_f = -\,W$ done by $\vec{E}$ in moving $+1\,C$ $(\infty \to f)$

- V due to apoint Charge:-

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$



$\uparrow E \uparrow dr$

Find $V_B - V_A$?

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r} \qquad \Rightarrow \qquad \vec{E} \cdot d\vec{r} = E\,dr$$

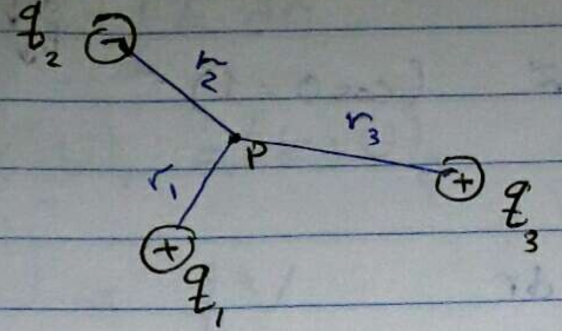
$$V_B - V_A = -\int_{r_A}^{r_B} \frac{q\,dr}{4\epsilon_0 \pi r^2} \Rightarrow V_B - V_A = \frac{-q}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{dr}{r^2}$$

$$\Rightarrow V_B - V_A = \frac{q}{4\pi\epsilon_0 r_B} - \frac{q}{4\pi\epsilon_0 r_A} \qquad \text{Volt}$$

$$V_B = \frac{q}{4\pi\epsilon_0 r_B} \qquad \text{Volt (let } A \to \infty)$$

- we must put the charge (+ or −)

• V due a set of point Charges :-



$$\Rightarrow V_1 = \frac{q_1}{4\pi\epsilon_0 r_1^2}$$

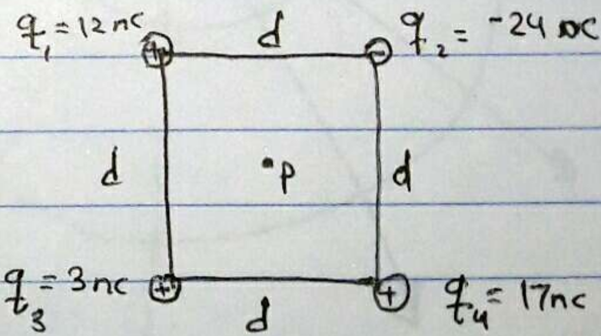
$$\Rightarrow V_2 = \frac{-q_2}{4\pi\epsilon_0 r_2^2}$$

$$\Rightarrow V_3 = \frac{q_3}{4\pi\epsilon_0 r_3^2}$$

$$V_P = V_1 + V_2 + V_3 \ldots$$

$$V_A = \sum V_i$$

• Sample problem <u>24.03</u>



$$r_1 = r_2 = r_3 = r_4 = \frac{d}{\sqrt{2}} = 0.92 \text{ m}$$

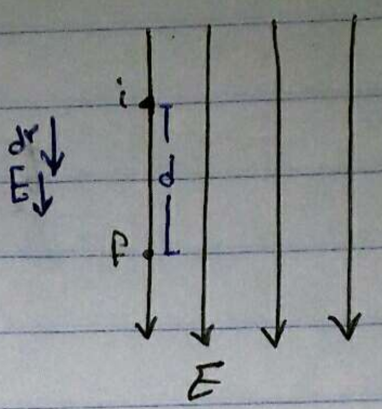
$$V_p = \sum \frac{q_i}{4\pi\epsilon_0 r_i}$$

$$= 9\times10^9 \left[\frac{12\times10^{-9}}{0.92} + \frac{31\times10^{-9}}{0.29} + \frac{17\times10^{-9}}{0.92} \right]$$

$$= 350 \text{ V}$$

- Sample problem 24.02:-

- Find $V_f - V_i$?

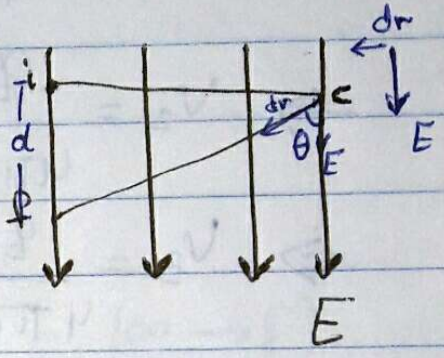


$$V_f - V_i = -\int_{r_i}^{r_f} \vec{E} . d\vec{r} \quad (\cos\theta = 1)$$

$$V_f - V_i = -E \int_{r_i}^{r_f} dr$$

$$V_f - V_i = -Ed$$

- Find $V_f - V_i$? throug path 2



$$V_f - V_i = -\int_{i}^{c} \vec{E} . d\vec{r} = 0 \quad (\cos 90 = zero)$$

$$V_f - V_c = -\int_{c}^{f} \vec{E} . d\vec{r}$$

$$V_f - V_c = -E\cos\theta \int_{c}^{f} d\vec{r}$$

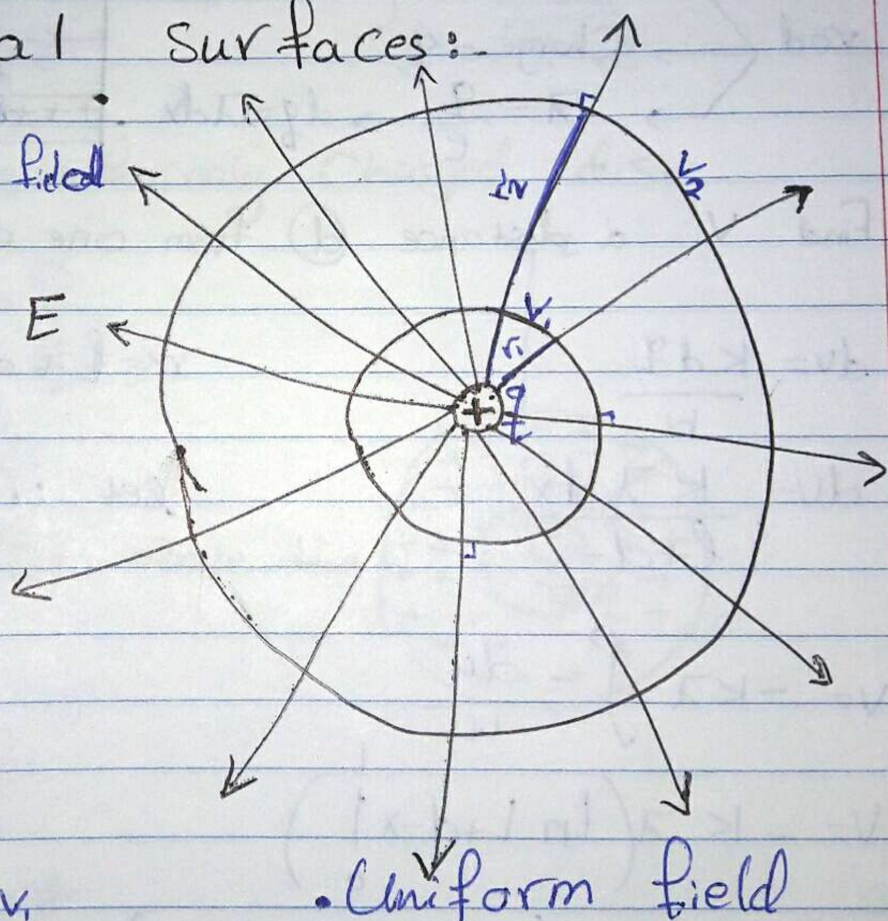
$$V_f - V_c = -E \frac{d}{l} l = -Ed$$

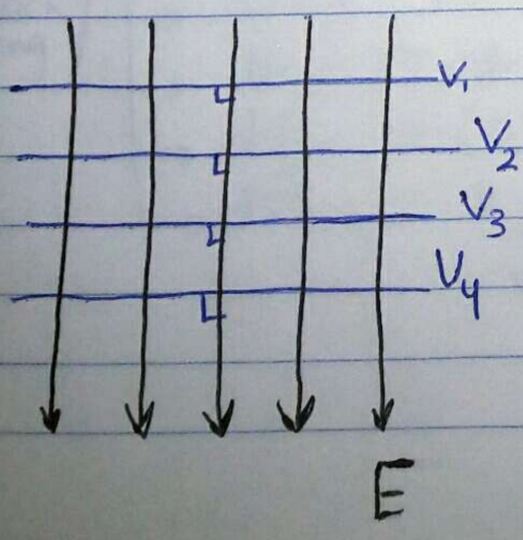
- $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{r}$
- $V_A = -\int_{\infty}^{A} E \cdot d\vec{r}$
- $(K+U)_i = (K+U)_f$
- $U = qV$
- $V = \dfrac{q}{4\pi\epsilon_0 r}$

*عند حساب الجهد نفترض وجود شحنة نقطية مقدارها ١ كولوم موجبة عند حساب طاقة الوضع فيجب وجود شحنة حقيقية.

• Equipotential surfaces:

- ununiform field
- $V_1 > V_2$

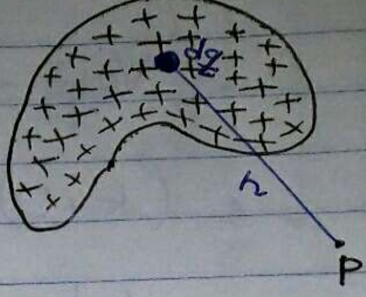




- Uniform field
- $V_1 > V_2 > V_3 > V_4$

• V due to a continuous Charge distribution:-

$$dv = \frac{k\,dq}{r}$$

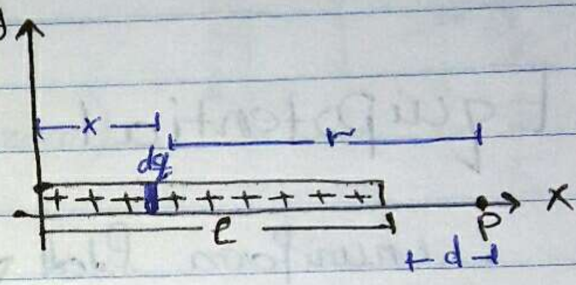


$$dv = k\frac{dq}{r}$$

$$V = \int dv$$

① V due to a uniformly charged rod:-

rod → length $= l$
→ Charge $= q$
→ $\lambda = \frac{q}{l} \rightarrow dq = \lambda dx$



• Find V a distance ⓓ from one end ?

$$dv = \frac{k\,dq}{r}$$

$r = l + d - x$

$$dv = \frac{k\lambda dx}{l+d-x}$$

let $u = l + d - x$

$$V = -k\lambda \int -\frac{du}{u}$$

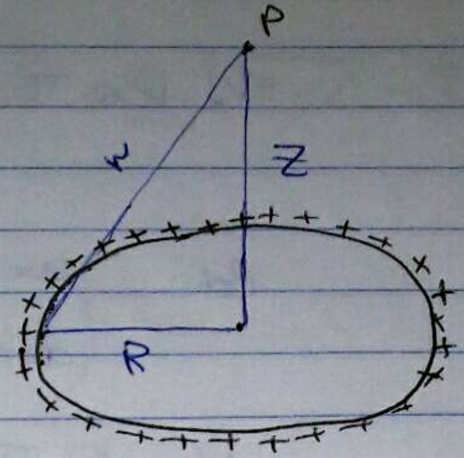
$$V = -k\lambda\left(\ln l+d-x \Big|_0^l\right)$$

$$= -k\lambda\,(\ln d - \ln l+d)$$

$$= -k\lambda\left(\ln \frac{d}{l+d}\right)$$

(2) V due to a uniformly Charged Ring:-

Ring → Radius = R
→ Charge = q
→ $\lambda = \frac{q}{2\pi R}$
→



. Find (V) at a point (z) above the Center ?

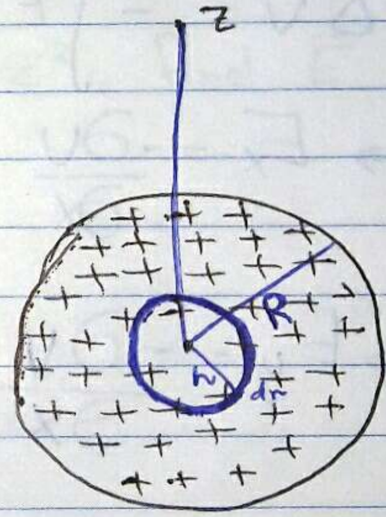
$$dV = \frac{k\,dq}{r}$$

$$\int dv = \int \frac{k\,dq}{\sqrt{R^2+Z^2}} \Rightarrow V_{ring} = \frac{Kq}{\sqrt{R^2+Z^2}}$$

(3) V due to a uniformly Charged disk:-

Disk → Radius = R
→ Charge = q
→ $\sigma = \frac{q}{\pi R^2}$

. Find V at a point (Z) above the center ?



divide the disk to rings:-

(each ring) → radius = r
→ width = dr
→ $dA = 2\pi r dr$
→ $dq = \sigma\,(2\pi r dr)$
→ $dV_{ring} = \frac{k\sigma 2\pi r\,dr}{\sqrt{r^2+z^2}}$

$$V_{disk} = \int dV_{ring}$$

$$= K \sigma \pi \int_0^R \frac{2r\,dr}{\sqrt{r^2+z^2}}$$

let $\quad u = r^2 + z^2$

$\quad du = 2r\,dr$

$$V = \frac{\sigma \pi}{4\epsilon_0} \int \frac{du}{u^{1/2}}$$

$$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2} - z\right)$$

· Calculating $\vec{E}$ from V :-

$$\Delta V = -\int \vec{E}\cdot d\vec{r} = -\int E_x dx - \int E_y dy - \int E_z dz$$

$\rightarrow E_x = -\dfrac{\partial V}{\partial x}$

$\rightarrow E_y = -\dfrac{\partial V}{\partial x}$

$\rightarrow E_z = -\dfrac{\partial V}{\partial z}$

• Sample problem 24.05

• Find $\vec{E}_{disk}$ from $V_{disk} = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+Z^2} - Z\right)$?

$$E_Z = -\frac{\partial V}{\partial Z}$$

$$E_Z = -\frac{\sigma}{2\epsilon_0}\left(\frac{2Z}{2\sqrt{R^2+Z^2}} - 1\right)$$

• problem : find E_{ring} from $V_{ring} = \frac{Kq}{\sqrt{R^2+Z^2}}$?

$$E_Z = -\frac{\partial V}{\partial Z}$$

$$E_Z = -Kq\left(\frac{2Z}{2} \sqrt{R^2+Z^2}\right)$$

• problem 24.17 , $V = 2xyZ^2$, find $\vec{E}$ at a point $\vec{r} = -\hat{i} - 2\hat{j} + 4\hat{k}$ m ??

$$E_x = -\frac{\partial V}{\partial x} = -2yZ^2$$

$$E_y = -\frac{\partial V}{\partial y} = -2xZ^2$$

$$E_Z = -\frac{\partial V}{\partial Z} = -2xy(2Z)$$

$$\left(\vec{E} = -2yZ^2\hat{i} - 2xZ^2\hat{j} - 4xyZ\hat{k}\right) V/m$$

$$\left(E = +16\hat{i} - 32\hat{j} - 32\hat{k}\right) V/m$$

- Electric potential energy of a system of Charged particles:-

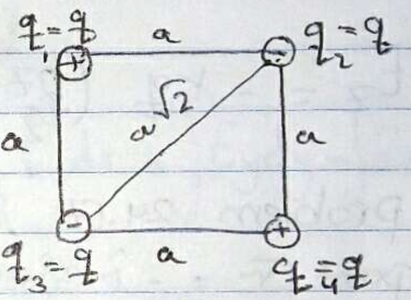
$$U = qV$$

$$W_E = -\Delta U = -q\,\Delta V$$

$$W_{F_{ext}} = \Delta U = q\,\Delta V$$

U = work done by external agent in moving each Charge q ($\infty \longrightarrow$ final position)

Problem $\boxed{33}$



(c)

$$U = w_1 + w_2 + w_3 + w_4$$

$$w_1 = 0$$

$$w_2 = q_2\left(\frac{kq_1}{a}\right)$$

$$w_3 = q_3\left(\frac{kq_1}{a} + \frac{kq_2}{a\sqrt{2}}\right)$$

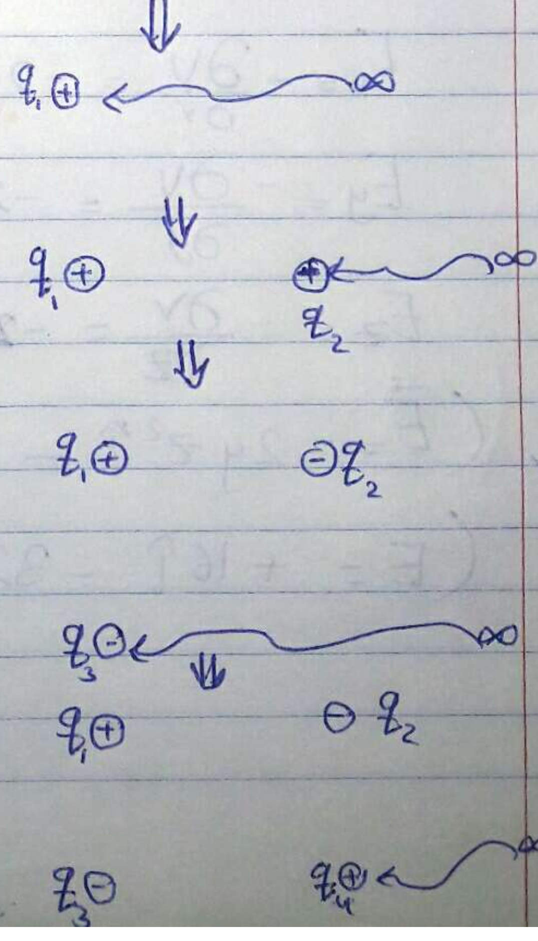
$$w_4 = q_4\left(\frac{kq_1}{a\sqrt{2}} + \frac{kq_2}{a} + \frac{kq_3}{a}\right)$$

$$U = \frac{k(-q)(q)}{a} + (-q)\left(\frac{kq}{a} + \frac{kq}{a\sqrt{2}}\right) + (q)\left(\frac{kq}{a\sqrt{2}} + \frac{kq}{a} - \frac{kq}{a}\right)$$

$$U = \frac{kq^2}{a}\left[-1 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - 1\right]$$

$$= \frac{kq^2}{a}\left[-4 + \frac{2}{\sqrt{2}}\right]$$

$$= -2.586\,\frac{kq^2}{a}\ J$$



ⓐ How much work done by external agent?

$$W_{ext} = \Delta U = -2.586 \frac{kq^2}{a}$$

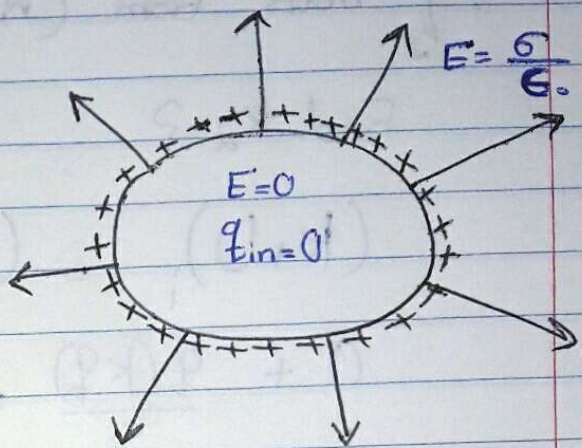
ⓑ $$W_E = -\Delta U = 2.586 \frac{kq^2}{a}$$

. Electric potential of charged isolated conducter:

$\Rightarrow \Delta V = \int \vec{E} \cdot d\vec{r}$

$\Rightarrow V_A - V_B = 0$

$\Rightarrow V_A = V_B = V_c$

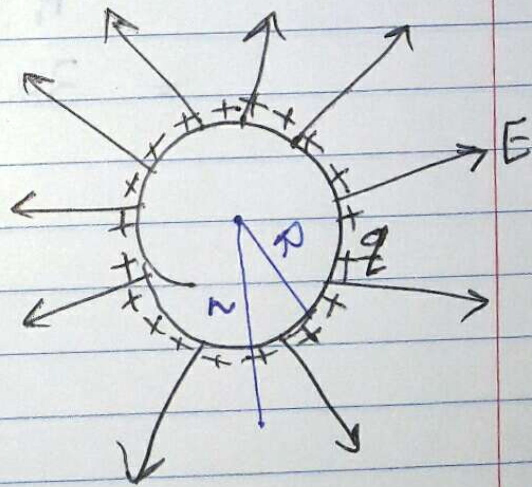


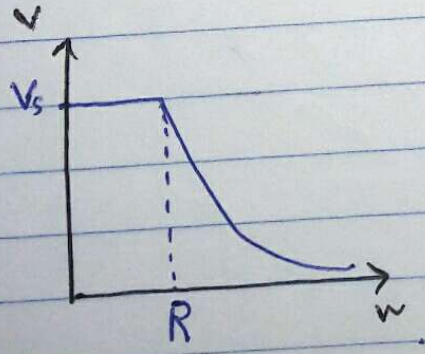
example:- Charged Conducting Sphere?

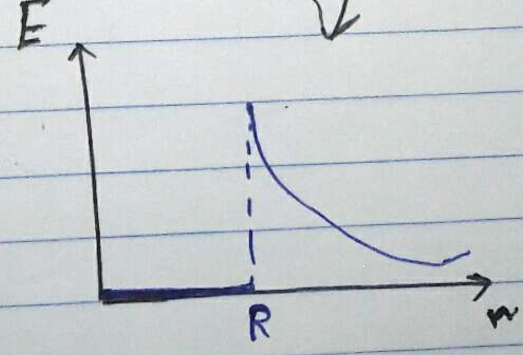
$\rightarrow V_{surface} = \frac{q}{4\pi\epsilon_0 R}$

$\rightarrow V_{surface} = V_{center} = \frac{q}{4\pi\epsilon_0 R}$

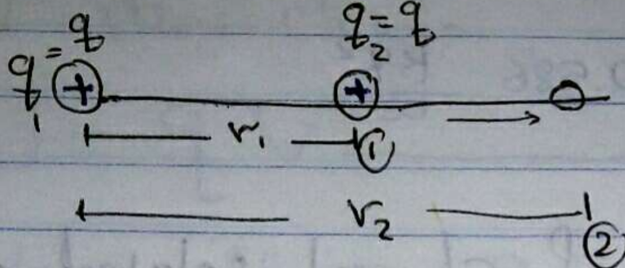
$\rightarrow V = \frac{q}{4\pi\epsilon_0 r}, \quad r > R$







• Problem <u>24-13</u>



• q_1 is fixed

• q_2 is free to move.

• q_2 moves from ($r_1 = 0.9\,mm \longrightarrow r_2 = 1.5\,mm$)

Find K_2?

$$(K+U)_i = (K+U)_f \qquad K_f = K_2$$

$$0 + \frac{q(kq)}{r_1} = K_2 + \frac{q(kq)}{r_2}$$

$$K_2 = \frac{kq^2}{r_1} - \frac{kq^2}{r_2}$$

$$= 38.4\,J$$